66-236

פתרון תרגיל 1

פתרון שאלה 2

1. $\Sigma(X_i-\bar{X})^2 = \Sigma(X_i^2-2X_i\bar{X}+\bar{X}^2) = \Sigma X_i^2 - 2\bar{X}\Sigma X_i + \Sigma\bar{X}^2 =$
$\Sigma X_i^2 - 2\bar{X}\cdot n\bar{X} + n\bar{X}^2 = \Sigma X_i^2 - n\bar{X}^2$

$\Sigma(x_i-\bar{x})^2 = \Sigma(x_i-\bar{x})(x_i-\bar{x}) = \Sigma x_i(x_i-\bar{x}) - \Sigma\bar{x}(x_i-\bar{x})$
$= \Sigma x_i(x_i-\bar{x}) - \bar{x}\underbrace{\Sigma(x_i-\bar{x})}_{0} = \Sigma x_i(x_i-\bar{x})$

2. $\Sigma(x_i-\bar{x})(y_i-\bar{y}) = \Sigma(x_i-\bar{x})y_i - \Sigma(x_i-\bar{x})\bar{y}$
$= \Sigma(x_i-\bar{x})y_i - \bar{y}\underbrace{\Sigma(x_i-\bar{x})}_{0}$

$\Sigma(x_i-\bar{x})(y_i-\bar{y}) = \Sigma x_i(y_i-\bar{y}) - \Sigma\bar{x}(y_i-\bar{y})$
$= \Sigma(y_i-\bar{y})x_i - \bar{x}\underbrace{\Sigma(y_i-\bar{y})}_{0}$

$\Sigma(x_i-\bar{x})(y_i-\bar{y}) = \Sigma x_i y_i - \Sigma x_i\bar{y} - \Sigma\bar{x}y_i + \Sigma\bar{x}\bar{y}$
$= \Sigma x_i y_i - \bar{y}\Sigma x_i - \bar{x}\Sigma y_i + n\bar{x}\bar{y}$
$= \Sigma x_i y_i - \bar{y}\cdot n\bar{x} - \bar{x}\cdot n\bar{y} + n\bar{x}\bar{y}$
$= \Sigma x_i y_i - n\bar{x}\bar{y}$

פתרון שאלה 3

1. $\bar{X} = \frac{\Sigma X}{n} = 10$
2. $\bar{Y} = \frac{\Sigma Y}{n} = 12$
3. $\bar{X}^2 = 10^2 = 100$
4. $\overline{X^2} = \frac{\Sigma X^2}{n} = 120$
5. $S_x^2 = \frac{\Sigma x^2}{n} - \bar{X}^2 = 20$
6. $S^2_{Y-1000} = S_Y^2 = \frac{\Sigma Y^2}{n} - \bar{Y}^2 = 16$
7. $S_{X,Y} = \frac{\Sigma XY}{n} - \bar{X}\bar{Y} = 16$
8. $S_{3X,-2Y} = 3\cdot|-2|\,S_{X,Y} = 96$
9. $S^2_{X+Y} = S^2_X + S^2_Y + 2S_{X,Y} = 20 + 16 + 2\cdot 16 = 68$
10. $S^2_{X-Y} = S^2_X + S^2_Y - 2S_{X,Y} = 20 + 16 - 2\cdot 16 = 4$

11. $\sigma^2_{3X-2Y} = \sigma^2_{3X} + \sigma^2_{2Y} + 2\cdot 3\cdot(-2)\sigma_{X,Y}$
$= 3^2 S_X^2 + 2^2 S_Y^2 + 2\cdot 3\cdot(-2)\cdot 16 =$
$= 3^2\cdot 20 + 2^2\cdot 16 + 2\cdot 3\,(-2)\cdot 16 = 52$

12. $\sigma^2_{3(X-Y)} = 3^2 S^2_{X-Y} = 3^2\cdot 4 = 36$

13. $\Sigma x^2 = \Sigma X^2 - n\bar{X}^2 = 600 - 5\cdot 10^2 = 100$

14. $\Sigma y^2 = \Sigma Y^2 - n\bar{Y}^2 = 800 - 5\cdot 12^2 = 80$

15. $\Sigma xy = \Sigma XY - n\bar{X}\bar{Y} = 680 - 5\cdot 10\cdot 12 = 80$

16. $\Sigma xX = \Sigma xx = \Sigma x^2 = 100$

17. $\Sigma xY = \Sigma xy = 80$

18. $r_{X,Y} = \dfrac{\Sigma xy}{\sqrt{\Sigma x^2\cdot \Sigma y^2}} = 0.8944$

19. $r^2_{X,Y} = 0.8$

20. $\sigma_{zx,zy} = 0.8944$

שאלה 4

1. $\bar{X} = 49.676$ $\quad \bar{Y} = 214.452$
$S_X = 11.535$ $\quad S_Y = 52.936$

2. $r_{X,Y} = 0.638$

שאלה 5

1. $\Sigma w_i = \Sigma \frac{x_i}{\Sigma x_i^2} = \frac{\Sigma x_i}{\Sigma x_i^2} = 0$

2. $\Sigma w_i^2 = \Sigma\left(\frac{x_i}{\Sigma x_i^2}\right)^2 = \frac{1}{(\Sigma x_i^2)^2}\cdot\Sigma(x_i)^2 = \frac{1}{\Sigma x_i^2}$

3. $\Sigma w_i x_i = \Sigma\left(\frac{x_i}{\Sigma x_i^2}\cdot x_i\right) = \Sigma\frac{x_i^2}{\Sigma x_i^2} = \frac{1}{\Sigma x_i^2}\cdot\Sigma x_i^2 = 1$

$\Sigma w_i x_i = \Sigma w_i (X_i - \bar{X}) = \Sigma w X_i - \Sigma w_i \bar{X} = \Sigma w_i X_i - \bar{X}\cancel{\Sigma w_i}^{\,0} = \Sigma w_i X_i$

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פתרון שאלה 6

א. $E(\bar{u}) = E\left(\frac{\sum u}{n}\right) = E\left(\frac{u_1+u_2+\ldots+u_n}{n}\right) = \frac{E(u_1)+E(u_2)+\ldots+E(u_n)}{n} = 0$

$$V(\bar{u}) = V\left(\frac{\sum u}{n}\right) = \frac{1}{n^2}V\left(\sum u\right) = \frac{1}{n^2}V(u_1+u_2+\ldots) =$$
$$= \frac{1}{n^2}\left[V(u_1)+V(u_2)+\ldots+2\,\mathrm{cov}(u_1,u_2)+\ldots\right]$$
$$= \frac{1}{n^2}\cdot nV(u) = \frac{V(u)}{n}$$

($\mathrm{cov}(u_1,u_2)$ crossed out, marked 0)

ב. $E(y) = E(x_1u_1+\ldots+x_nu_n) = E(x_1u_1)+\ldots+E(x_nu_n) = x_1E(u_1)+\ldots+x_nE(u_n) = 0$

$$V(y) = V(x_1u_1+\ldots+x_nu_n) = V(x_1u_1)+\ldots+V(x_nu_n)+2\,\mathrm{cov}(x_1u_1,x_2u_2)+\ldots$$
$$= x_1^2V(u_1)+\ldots+x_n^2V(u_n)+2x_1x_2\,\mathrm{cov}(u_1,u_2)+\ldots$$
$$= x_1^2V(u)+\ldots+x_n^2V(u) = V(u)\sum x_i^2$$

ג. מכפלה של קבוע במ"מ נורמלי — מתפלג נורמלית

סכום של מ"מ נורמליים — מתפלג נורמלית

מכיון, שכל u_i מתפלג נורמלית — גם Y מתפלג נורמלית