פתרון שאלה 1

א.

$$Y = X\beta + u$$
$$E(u) = 0 \qquad V(u) = \sigma^2 I \qquad E(X'u) = 0$$

ק.

1. $X'X = \begin{pmatrix} 5 & 15 & 25 \\ 15 & 55 & 81 \\ 25 & 81 & 129 \end{pmatrix}$

2. $x'x = \begin{pmatrix} 10 & 6 \\ 6 & 4 \end{pmatrix}$

3. $X'Y = \begin{pmatrix} 20 \\ 76 \\ 109 \end{pmatrix}$

4. $x'y = \begin{pmatrix} 16 \\ 9 \end{pmatrix}$

5. $Y'Y = 108$

6. $y'y = 28$

7. $(X'X)^{-1} = \begin{pmatrix} 26.7 & 4.5 & -8 \\ 4.5 & 1 & -1.5 \\ -8 & -1.5 & 2.5 \end{pmatrix}$

8. $(x'x)^{-1} = \begin{pmatrix} 1 & -1.5 \\ -1.5 & 2.5 \end{pmatrix}$

9. $i'X = (\begin{matrix} 5 & 15 & 25 \end{matrix})$

10. $i'x = (\begin{matrix} 0 & 0 \end{matrix})$

$$b = (X'X)^{-1}X'Y = \begin{pmatrix} 4 \\ 2.5 \\ -1.5 \end{pmatrix} \qquad \qquad \text{ج.}$$

$$(X'X)\,b = \begin{pmatrix} 20 \\ 76 \\ 109 \end{pmatrix} \qquad X'Y = \begin{pmatrix} 20 \\ 76 \\ 109 \end{pmatrix} \qquad \text{3.}$$

$$(x'x)b = \begin{pmatrix} 16 \\ 9 \end{pmatrix} \qquad x'y = \begin{pmatrix} 16 \\ 9 \end{pmatrix} \qquad \text{د.}$$

Y	X_2	X_3	$\hat{Y}=4+2.5X_2-1.5X_3$	e	e^2	.1
3	3	5	4	−1	1	
1	1	4	0.5	0.5	0.25	
8	5	6	7.5	0.5	0.25	
3	2	4	3	0	0	
5	4	6	5	$\frac{0}{0}$	$\frac{0}{1.5}$	

$$\Sigma e = 0 \qquad \Sigma e^2 = 1.5 \qquad S^2 = \frac{1.5}{5-3} = 0.75 \qquad \text{ s.}$$

$$e'e = 1.5 \qquad y'y = 28 \qquad b'x'y = \begin{pmatrix} 2.5 & -1.5 \end{pmatrix}\begin{pmatrix} 16 \\ 9 \end{pmatrix} = 26.5 \qquad \text{n.}$$

$$R^2 = 1 - \frac{e'e}{y'y} = 1 - \frac{1.5}{28} = 0.9464 \qquad \text{ v.}$$

$$e = Y - Xb = Y - X \cdot (X'X)^{-1}X'Y = \left(I - X(X'X)^{-1}X' \right) Y \qquad \text{.}$$

$$M = I - X(X'X)^{-1}X' \qquad \text{.1}$$

$$M \cdot M = M \qquad \text{ .2}$$

$$M' = M \qquad \text{ .3}$$

$$e'e = (MY)'MY = Y'\underset{I}{M'M}Y = Y'MY \geq 0 \qquad \text{ .4}$$

$$MX = (I - X(X'X)^{-1}X')X = X - X(X'X)^{-1}X'X = 0 \qquad \text{ .5}$$

$$M = \begin{pmatrix} 0.8 & -0.2 & 0.2 & -0.2 & -0.2 \\ -0.2 & 0.3 & 0.3 & -0.2 & -0.2 \\ -0.2 & 0.3 & 0.3 & -0.2 & -0.2 \\ -0.2 & -0.2 & -0.2 & 0.3 & 0.3 \\ -0.2 & -0.2 & -0.2 & 0.3 & 0.3 \end{pmatrix} \quad .6$$

$$\text{trace}(M) = 0.8 + 0.3 + 0.3 + 0.3 + 0.3 = 2 \quad .7$$
$$\text{trace}(M) = n - k$$

1. $e'e = Y'Y - b'X'Y$

$$e'e = (Y - Xb)'(Y - Xb) = (Y' - b'X')(Y - Xb)$$

$$= Y'Y - Y'Xb - b'XY + b'X'Xb$$

$$= Y'Y - b'X'Y + (\underbrace{b'X'Xb - Y'Xb}_{})$$

$b = (X'X)^{-1}X'Y$ (נראה 3)

וזה נתון שווה לאפס.

2. $e'e = y'y - b'x'y$

$$e'e = (y - xb)'(y - xb) = y'y - y'xb - b'x'y + b'x'xb$$

$$= y'y - b'x'y + (\underbrace{b'x'xb - y'xb}_{})$$

$b = (x'x)^{-1}x'y$, (נראה 3)

וזה נתון שווה לאפס.

3. $R^2 = \dfrac{b'x'Y - n\bar{Y}^2}{Y'Y}$

$$R^2 = 1 - \frac{e'e}{y'y} = \frac{y'y - e'e}{y'y} = \frac{y'y - n\bar{y}^2 - (y'y - b'x'y)}{y'y} = \frac{b'x'y - n\bar{y}^2}{y'y}$$

4. $R^2 = \dfrac{b'x'y}{y'y}$

$$R^2 = 1 - \frac{e'e}{y'y} = \frac{y'y - e'e}{y'y} = \frac{y'y - (y'y - b'x'y)}{y'y} = \frac{b'x'y}{y'y}$$

5. $e'e = u'Mu$

$e = Y - Xb = (X\beta + u) - X \cdot (x'x)^{-1}x'(X\beta + u) =$

$= X\beta + u - x(x'x)^{-1}x'x\beta - x(x'x)^{-1}x'u = [I - x(x'x)^{-1}x']u = Mu$

$e'e = (Mu)'Mu = u'M'Mu = u'Mu$